Objective: Develop and Apply the Pythagorean Identities

**Pythagorean Identities**

You can use the Pythagorean Theorem with the unit circle to derive an important identity called the Pythagorean Identity.

Consider the right triangle with $x = \cos \theta$ and $y = \sin \theta$.

Since the radius is 1, we have:

\[
\cos^2 \theta + \sin^2 \theta = 1
\]

The Pythagorean Identity is often written as $\sin^2 \theta + \cos^2 \theta = 1$. You can easily derive the following Pythagorean Identities.

\[
\sec^2 \theta = 1 + \tan^2 \theta
\]

\[
\csc^2 \theta = 1 + \cot^2 \theta
\]

You can use your graphics calculator to visualize the Pythagorean Identities.
Applying the Pythagorean Identities

You can use Pythagorean Identities to solve simple problems.

Suppose \( \tan \theta = 4 \). Find \( \sin \theta \) and \( \cos \theta \) with \( \cos \theta > 0 \).

First solve for \( \sec \theta \) using the identity \( \sec^2 \theta = 1 + \tan^2 \theta \).

\[
\sec^2 \theta = 1 + \tan^2 \theta
\]
\[
\sec^2 \theta = 1 + 4^2
\]
\[
\sec \theta = \pm \sqrt{17}
\]

Since \( \cos \theta > 0 \), we have:

\[
\sec \theta = \sqrt{17}
\]
\[
\frac{1}{\cos \theta} = \sqrt{17}
\]
\[
\cos \theta = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}
\]

Now use the tangent identity to solve for sine.

\[
\frac{\sin \theta}{\cos \theta} = \tan \theta
\]
\[
\sin \theta = \tan \theta \cos \theta
\]
\[
\sin \theta = 4 \left( \frac{\sqrt{17}}{17} \right) = \frac{4\sqrt{17}}{17}
\]

You can check with your calculator:

\[
\cos(\tan^{-1}(4)) = 0.242535625
\]
\[
\sin(\tan^{-1}(4)) = 0.9701425001
\]
\[
\sqrt{17}/17 = 0.242535625
\]
\[
4\sqrt{17}/17 = 0.9701425001
\]