The Trigonometric Functions

Each of the trigonometric functions—sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot)—is a ratio of sides in a right triangle.

Given the right triangle shown, we can define the trigonometric functions as follows:

\[
\begin{align*}
\sin A &= \frac{a}{c} & \sin B &= \frac{b}{c} \\
\cos A &= \frac{b}{c} & \cos B &= \frac{a}{c} \\
\tan A &= \frac{a}{b} & \tan B &= \frac{b}{a} \\
\csc A &= \frac{c}{a} & \csc B &= \frac{c}{b} \\
\sec A &= \frac{c}{b} & \sec B &= \frac{c}{a} \\
\cot A &= \frac{b}{a} & \cot B &= \frac{a}{b}
\end{align*}
\]

Note the following equalities:

\[
\begin{align*}
\sin A &= \cos B, \quad \cos A = \sin B, \quad \tan A = \cot B, \quad \csc A = \sec B, \quad \sec A = \csc B, \quad \text{and} \quad \cot A = \tan B
\end{align*}
\]
Trigonometric Values of Special Right Triangles

You saw that a 45-45-90 right triangle has side lengths $s$, $s$, and $s\sqrt{2}$. To compute trigonometric values for the 45 angle, consider the right triangle with side length 1. So the hypotenuse has length $\sqrt{2}$.

Now we can easily compute trigonometric values.

\[
\begin{align*}
\sin 45 &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \\
\cos 45 &= \frac{\sqrt{2}}{2}, \\
\tan 45 &= 1
\end{align*}
\]

You saw that a 30-60-90 right triangle has side lengths $s$, $s\sqrt{3}$, and $2s$. To compute trigonometric values for the 30 and 60 degree angles, consider the right triangle with short side length 1.

So the hypotenuse has length 2 and the long leg has length $\sqrt{3}$.

\[
\begin{align*}
\sin 30 &= \frac{1}{2}, \\
\cos 30 &= \frac{\sqrt{3}}{2}, \text{ and } \\
\tan 30 &= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.
\end{align*}
\]

\[
\begin{align*}
\sin 60 &= \frac{\sqrt{3}}{2}, \\
\cos 60 &= \frac{1}{2}, \text{ and } \\
\tan 60 &= \frac{\sqrt{3}}{1} = \sqrt{3}.
\end{align*}
\]