Objective: Determine Average Rates of Change

Computing Average Rates of Change

You have already seen how to compute the rate of change, or slope of a line.

Given any two points on a line, \((x_1, y_1)\) and \((x_2, y_2)\),

\[
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.
\]

You can compute average rates of change for other functions using your calculator and your knowledge of slope.

Consider the function, \(h(x) = -16x^2 + 80x\), where \(h(x)\) is the height of an object in feet and \(x\) is time in seconds. You can approximate the speed at 2 second by computing the average rate of change between 2 and 2.1 seconds. You can see from the graph that the rate of change is positive. Using the Trace feature of your calculator you can now compute the average rate of change, or speed of the object from 2 to 2.1 seconds.

\[
\Delta y = \frac{97.44 - 96}{2.1 - 2} = \frac{1.44}{0.1} = 14.4 \text{ feet/second}
\]
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Computing Rates of Change of the Form \( \frac{f(x+h) - f(x)}{h} \)

Consider the function, 
\( h(x) = -16x^2 + 80x \), where \( h(x) \) is the height of an object in feet and \( x \) is time in seconds. To better approximate the speed at 2 seconds you can compute the rate of change between 2 and 2.01 seconds.

\[
\begin{array}{c|c|c}
 x & \text{Y1} \\
 2.00 & 56.1584 \\
 2.01 & 64.0196 \\
\end{array}
\]

\( Y1=96.1584 \)

At \( x=2 \) and any small number, \( h \), we approximate the rate of change between 2 and \( 2+h \) second. We have:

\[
\frac{\Delta y}{\Delta x} = \frac{f(2.01) - f(2)}{2.01 - 2} = \frac{f(2 + .01) - f(2)}{0.1} = \frac{96.1584 - 96}{.01} = \frac{.1584}{.01} = 15.84 \text{ ft/sec}
\]

\[
\frac{\Delta y}{\Delta x} = \frac{f(2 + h) - f(2)}{h} = \frac{-16(2 + h)^2 + 80(2 + h) - (-16(2)^2 + 80(2))}{h} = \frac{-64 - 64h - 16h^2 + 160 + 80h + 64 - 160}{h} = \frac{-16h^2 + 16h}{h} = -16h + 16
\]
More generally for any time, $x$, and any non-zero $h$ we have:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{f(x+h) - f(x)}{h}$$

$$= \frac{-16(x+h)^2 + 80(x+h) - (-16x^2 + 80x)}{h}$$

$$= \frac{-16x^2 - 32xh - 16h^2 + 80x + 80h + 16x^2 - 80x}{h}$$

$$= \frac{-16h^2 - 32xh + 80h}{h}$$

$$= -16h - 32x + 80$$

Recall:

$$(x+h)^2 = x^2 + 2xh + h^2$$
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