Unit 1 Functions
1.1 Evaluate and graph relations and functions
1.1.1 Determine whether or not a relation is a function.

Example: Determine if the relation is a function.

1. Year Population
   0  10000
   1  9500
   2  9025
   3  8574
   4  8145
   5  7738

Try This: Determine if the relation is a function.

1. Grams Length
   10  21
   20  39
   15  32
   10  19
   5  9
   30  61

3. $x^2 + y^2 = 25$
1.1.2 Build a table and graph for each function.

Example:
1. An object is projected straight up into the air from the ground with an initial velocity of 80\(\text{ft/s}\). Build a table and graph for \(h(x) = -16x^2 + 80x\) and interpret the meaning.

2. The population growth of a community is controlled with the equation \(P(x) = 12000\sqrt{0.5x+1}\), where \(P(x)\) is population and \(x\) is time in years beginning with \(x=0\). Build a table and graph for \(P(x)\) and interpret the meaning.

Try This:
1. The cost of a rental is given by \(C(x) = 2.25x + 3.75\) where \(C\) is cost and \(x\) is time in hours. Build a table and graph for \(C(x)\) and interpret the meaning.

2. Build a table and graph for \(f(x) = x^3 + 3x^2 - 4x - 12\).

3. Build a table and graph for \(g(x) = \frac{x^2 - 1}{x^2 - x - 6}\).
1.1.3 Identify the domain and range of a function.

Example: Determine the domain and range for each function.

1. \( h(x) = -16x^2 + 80x \)
2. \( f(x) = \sqrt{2x - 8} \)
3. \( r(x) = \frac{x^2 - 3}{x^2 + 4x + 4} \)

Try This: Determine the domain and range for each function.

1. \( s(x) = \sqrt{1 - x^2} \)
2. \( f(x) = x^3 + 3x^2 - 4x - 12 \)
3. \( g(x) = \frac{x^2 - 1}{x^2 - x - 6} \)

1.2 Evaluate functions to determine average rates of change

1.2.1 Given a function determine the average rate of change between two points.

Example:

1. For \( h(x) = -16x^2 + 80x \) where \( h \) is height in feet and \( x \) is time in seconds, find the average velocity of the object between time 1 and time 2.

2. For \( h(x) = -16x^2 + 80x \) find the average velocity of the object between time 1 and time 1.1.

Try This:

1. For \( f(x) = 2x + 1 \) find the average rate of change between \( x = 2 \) and \( x = 3 \).
2. For \( g(x) = x^2 - 1 \) find the average rate of change between \( x = 2 \) and \( x = 2.1 \).
3. Find the average rate of change for \( p(x) = \sqrt{x} + 2 \) between \( x = 0 \) and \( x = 1 \).
1.2.2 Given a function compute and simplify \( \frac{f(x+h) - f(x)}{h} \).

Example:

<table>
<thead>
<tr>
<th>1. For ( f(x) = -16x^2 + 80x ) find ( f(1+h) ) and ( \frac{f(1+h) - f(1)}{h} ).</th>
<th>2. For ( f(x) = -16x^2 + 80x ) find ( \frac{f(x+h) - f(x)}{h} ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. For ( f(x) = 2x + 1 ) find ( \frac{f(2+h) - f(2)}{h} ).</td>
<td>2. For ( f(x) = x^2 - 1 ) find ( \frac{f(x+h) - f(x)}{h} ).</td>
</tr>
</tbody>
</table>

Try This:

1. For \( f(x) = 2x + 1 \) find \( \frac{f(2+h) - f(2)}{h} \).
2. For \( f(x) = x^2 - 1 \) find \( \frac{f(x+h) - f(x)}{h} \). 
3. For \( f(x) = \sqrt{x+2} \) find \( \frac{f(x+h) - f(x)}{h} \).

1.3 Solve equations related to functions

1.3.1 Given an output, solve for the input algebraically.

Example:

1. The height of a bottle rocket is modeled with the function \( h(x) = -16x^2 + 80x \). When is it at a height of 96 ft?

2. The population growth of a community is controlled with the equation \( P(x) = 12000\sqrt{0.5x + 1} \), where \( P(x) \) is population and \( x \) is time in years beginning with \( x = 0 \). When does the population double?
Try This:

1. Given \( f(x) = x^{3/2} \), for what value of \( x \) is \( f(x) = 125 \)?

2. Given \( f(x) = x^4 - 5x^2 \), for what value(s) of \( x \) is \( f(x) = -6 \)?

1.3.2 Given an output solve for the input graphically.

Example:

1. The height of an object is modeled with the function \( h(x) = -16x^2 + 80x \), where \( x \) is time in seconds. When is it at a height of 80 ft?

2. The population growth of a community is controlled with the equation 
   \[ P(x) = 12000 \sqrt{0.5x + 1} \] where \( P(x) \) is population and \( x \) is time in years beginning with \( x = 0 \). When does the Population reach 20,000?

Try This:

1. Given \( f(x) = \frac{20x}{x^2 + 1} \), for what value of \( x \) is \( f(x) = 10 \)?

2. The volume of a container is given by \( V(x) = x^3 - 14x^2 + 48x \). For what value(s) of \( x \) is the volume 42?

1.4 Perform operations on Functions.
1.4.1 Perform basic operations on functions.

Examples:

1. For \( f(x) = x^2 + 6x + 9 \) and \( g(x) = 4x - 8 \), compute \( (f + g)(x) \), \( (f - g)(x) \), \( (f \cdot g)(x) \) and \( (f / g)(x) \).
2. For \( f(x) = \frac{x}{2x-1} \) and \( g(x) = \frac{x-8}{x+2} \), compute \( (f + g)(x) \), \( (f - g)(x) \), \( (f \cdot g)(x) \) and \( (f / g)(x) \).

Try This:
1. For \( f(x) = x^2 - 9 \) and \( g(x) = x^2 - x - 9 \), compute \( (f + g)(x) \), \( (f - g)(x) \), \( (f \cdot g)(x) \) and \( (f / g)(x) \).

2. For \( f(x) = \frac{3x-5}{x-3} \) and, compute \( g(x) = \frac{x}{x^2-3} \) \( (f + g)(x) \), \( (f - g)(x) \), \( (f \cdot g)(x) \) and \( (f / g)(x) \).

1.4.2 Given two functions, \( f \) and \( g \), find and compare \( (f \circ g)(x) \) and \( (g \circ f)(x) \).

Example:
1. The height of a bottle rocket is modeled with the function \( h(x) = -16x^2 + 80x \). Consider the delay function \( d(x) = x - 2 \). Compute \( h \circ d(x) \) and interpret numerically and graphically.
2. The height of a bottle rocket is modeled with the function \( h(x) = -16x^2 + 80x \). Consider the kick function \( k(x) = x + 96 \). Compute \( k \circ h(x) \) and interpret numerically and graphically.

Try This:

1. For \( f(x) = x^2 + 6x + 9 \) and \( g(x) = 4x - 8 \), compute both \( f \circ g(x) \) and \( g \circ f(x) \). How do they compare?

2. For \( f(x) = 2x - 5 \) and \( g(x) = \frac{x + 5}{2} \), compute both \( f \circ g(x) \) and \( g \circ f(x) \). How do they compare?

1.4.3 Given a function write it as the composition of two functions.

Example:

<table>
<thead>
<tr>
<th>1. For ( h(x) = \sqrt{2x + 5} ) find two functions so that ( f \circ g(x) = h(x) ).</th>
<th>2. For ( c(x) = (x - h)^3 + k ) find two functions so that ( f \circ g(x) = c(x) ).</th>
</tr>
</thead>
</table>

Try This:

<table>
<thead>
<tr>
<th>1. For ( h(x) = \left(\sqrt{x}\right)^3 ) find two functions so that ( f \circ g(x) = h(x) ).</th>
<th>2. For ( c(x) = a(x - h)^3 + k ) find two functions so that ( f \circ g(x) = c(x) ).</th>
</tr>
</thead>
</table>
1.5 Given a function, find its inverse
1.5.1 Identify the inverse of a function numerically and graphically.

Example: For the given relation, graph and connect the points with a smooth curve. Record the inverse relation in the blank table. Then graph and connect those points with a smooth curve.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Try This:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
</tr>
</tbody>
</table>

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</tr>
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<tr>
<td>-1</td>
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</tr>
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<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>
1.5.2 Given a table, graph or rule for a function, determine whether or not its inverse is a function.

Example: For the given function, determine if it has an inverse function.

1. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-1 & 0.5 \\
0 & 1 \\
1 & 2 \\
2 & 4 \\
3 & 8 \\
\hline
\end{array}
\]

2. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-2 & 9 \\
-1 & 4 \\
0 & 1 \\
1 & 0 \\
2 & 1 \\
3 & 4 \\
\hline
\end{array}
\]

3. \( y = (x - 3)^3 \)

Try This: For the given function, determine if it has an inverse function.

1. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-1 & 0.5 \\
0 & 1 \\
1 & 2 \\
2 & 4 \\
3 & 8 \\
\hline
\end{array}
\]

2. \[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
-2 & 9 \\
-1 & 4 \\
0 & 1 \\
1 & 0 \\
2 & 1 \\
3 & 4 \\
\hline
\end{array}
\]

3. \( y = x^4 - x^2 \)
1.5.3 Given $f$ determine $f^{-1}$ and compose $f$ with $f^{-1}$ and $f^{-1}$ with $f$.

Example:

1. Zero degrees Celsius is equal to 32 degrees Fahrenheit and 100 degrees Celsius is equal to 212 degrees Fahrenheit. Write $F$ as a function of $C$. Then find $C$ as a function of $F$ two ways.

Try This:

1. For $f(x) = \frac{2x-3}{4}$ find $f^{-1}(x)$.

2. For $f(x) = -\frac{3}{4}x + 7$ find $f^{-1}(x)$.

3. For $f(x) = 3x^3 + 1$ find $f^{-1}(x)$. 