Solve Quadratics Inequalities

Consider the function,

\[ f(x) = x^2 - x - 6 = (x + 2)(x - 3), \]

with zeros at \( x = -2, x = 3 \).

It is clear from the graph that the solution to the inequality \( x^2 - x - 6 > 0 \) is all real numbers \( x \), such that \( x < -2 \) or \( x > 3 \).

It is also clear from the graph that the solution to the inequality \( x^2 - x - 6 < 0 \) is all real numbers \( x \), such that \( -2 < x < 3 \)

The following statement generalizes these two ideas.

Consider the function, \( f(x) = ax^2 + bx + c \), with distinct real zeros at \( x_1, x_2 \) with \( x_1 < x_2 \).

The solution to the quadratic inequality \( ax^2 + bx + c > 0 \), with \( a > 0 \) is the set of all real numbers, \( x \), such that \( x < x_1 \) or \( x > x_2 \).

The solution to the quadratic inequality \( ax^2 + bx + c < 0 \), with \( a > 0 \) is the set of all real numbers, \( x \), such that \( x_1 < x < x_2 \).
Solve Quadratic Application Inequalities

The height, \( h(t) \), of an object where \( h(t) \) is feet and \( t \) is time in seconds is given by \( h(t) = h_0 + v_0 t - 0.5(32)t^2 \) where \( h_0 \) is the initial height, \( v_0 \) is the initial velocity in ft/sec, and 32 ft/sec\(^2\) is acceleration due to gravity.

The height of an object is given by the function

\[
h(t) = 6 + 120t - 0.5(32)t^2.
\]

When is the object at a height of at least 200 ft (to the nearest tenth of a second)?

Solving graphically we have:

\[
2.4 \leq t \leq 5.1
\]