Write and graph the quadratic function in vertex form and determine the domain and range, and intervals over which the function is increasing or decreasing.

We are given: \( f(x) = -2x^2 - 10x + 7 \)

In order to rewrite the given function into vertex form, we use the technique of completing the square.

Since the coefficient of the \( x^2 \) term is -2, we need to factor -2 from the variable terms.

\[
f(x) = -2(x^2 + 5x) + 7
\]

In order to complete the square, we are going to manipulate the terms into a quantity squared in the form of \((x + h)^2\). We do this by taking half of the coefficient in front of the \( x \) term is and squaring that number and adding it to our function. We also must subtract it to maintain equality.

\[
f(x) = -2\left(x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right) + 7
\]

We next want to isolate and factor the perfect square trinomial:

\[
f(x) = -2\left(x^2 + 5x + \frac{25}{4}\right) - 2\left(-\frac{25}{4}\right) + 7 = -2\left(x^2 + 5x + \frac{25}{4}\right) + \frac{39}{2}
\]

\[
f(x) = -2\left(x + \frac{5}{2}\right)^2 + \frac{39}{2}
\]

To sketch the graph, we will begin with the vertex which can be read off our vertex-form quadratic as \((-\frac{5}{2}, \frac{39}{2})\) and then we can make a list of \(x\)-\(y\) values of the function on both sides of \(-\frac{5}{2}\) to give us points that we can use to sketch our parabola.

Since the function has no discontinuity for any number \(x\), the domain of the function is negative infinity to positive infinity, or all real numbers: \((-\infty, \infty)\). The range of the function will be the \(y\)-value of the vertex (which will be either the upper or lower limit in the range) and either go on to positive or negative infinity depending on which way the parabola opens. Since the parabola opens down, \(\frac{39}{2}\) will be our upper limit of our range, and the lower limit will be negative infinity: \((-\infty, \frac{39}{2}]\).

Finally, we want to find the intervals over which the function is increasing or decreasing. Since the parabola opens down, the function is increasing between negative infinity and the \(x\)-value of
the vertex. Thus the interval of increase is $(-\infty, -\frac{5}{2})$. Likewise, since the $y$-values of the function are decreasing from left to right from the vertex and onward, the interval of decrease is $(-\frac{5}{2}, \infty)$. 